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New bounds for the torus variant of the no-three-in-a-line problem

The torus variant of the no-three-in-a-line problem seeks to determine the maximum amount of points that can be placed on a discrete $n \times m$ torus such that no three points lie in a line. We denote such a quantity by $\tau(n, m)$. In the torus, lines are the projection of lines in $\mathbb{Z}^2$, where each point (x, y) projects to $(x \mod n, y \mod m)$. The problem has been solved for tori $\mathbb{Z}_n \times \mathbb{Z}_m$ where m and n are coprime or $gcd(m, n) = p$ is a prime [**?**], however only trivial bound are known for square tori with composite dimensions.

In this work we give improved bounds for square tori $\mathbb{Z}_n^2$. We present a lower bound construction when $n = p^2$ for prime p, proving $\tau(p^2, p^2) \geq 2p$. We also show $p + 4 \leq \tau(2p, 2p) \leq \frac{88}{45}p + \frac{16}{9}$, improving the previous bounds of $p + 1 \leq \tau_{2p} \leq 2p + 2$ and solving a conjecture from [**?**]. Finally, we show that $\tau(n, n) = \Omega\left(\sqrt{\frac{n}{\log \log n}}\right)$ for every $n \geq 3$.

This is joint work with Brian Hearn and Cameron Strachan.